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Trajectory Estimation for Satellite Clusters

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The problem of tracking a satellite cluster by using the periodic orbit/Floquet theory solution to relative motion can be separated into two coupled estimation problems: determining the periodic orbit, and determining the relative motion of the cluster. We have studied this problem with simulated global positioning system (GPS) data for the periodic orbit estimator and both simulated differential GPS data and timing synchronization data for the relative problem. After careful treatment of several singularities and instabilities, we have achieved accuracy of about 1 m in determining the periodic reference orbit and about 2-cm error in the relative motion. The relative motion estimate is very insensitive to significantly mismatched dynamics (e.g., solar flares) that do significantly degrade the estimate of the reference orbit.

Nomenclature

$\boldsymbol{a}_p$	=	perturbing acceleration vector
B	=	ballistic coefficient
$\boldsymbol{D}$	=	generic data vector
E	=	Floquet modal matrix
$\boldsymbol{\mathcal{I}}$	=	inertial frame state vector
$\boldsymbol{\mathcal{I}}_{\mathrm{PO}}$	=	interial frame periodic orbit state vector
$\mathcal{I}_{\mathrm{PO,pert}}$	=	forced perturbations to the periodic orbit
i_0	=	initial orbital inclination
$\mathcal{J}$	=	Jordan form of Poincaré exponents
$\mathcal{N}$	=	nodal frame representation of the periodic orbit
P	=	state covariance matrix
Q	=	least-squares data covariance matrix
$R_Z^{(2)}$	=	z-axis precession rotation matrix
$\boldsymbol{r}$	=	data residual vector
T	=	least-squares observation sensitivity matrix
τ	=	period of the periodic orbit
X, Y, Z	=	inertial position components
$\boldsymbol{z}$	=	modal state vector
$\delta\boldsymbol{X}$	=	least-squares state correction
Λ	=	second-order modal perturbation tensor
ϕ	=	orbit phase
Ω_0	=	right ascension of the ascending node
$\dot{\Omega}$	=	nodal regression rate

Introduction

CLUSTERS of satellites have been proposed for several missions, including phased-array radars and radio direction finding in orbit. These applications will require extremely accurate relative motion determination. Furthermore, to keep stationkeeping requirements as small as possible, the dynamics must carefully separate benign perturbations from those effects that can disperse the cluster. In this effort we will employ the solution of Wiesel,[1] which uses a nearly circular periodic orbit and its local Floquet solution to model the relative dynamics. We will combine this with simulated data from global positioning system (GPS), differential GPS, and ranging between the satellites in the cluster to determine the motion of the system.

One of the most important considerations in this effort will be to separate the problem of estimating the baseline periodic orbit from the problem of estimating the cluster satellite's individual relative motions. This is motivated by the fact that subtracting individual satellites' absolute positions to obtain their relative position introduces a very severe loss of accuracy. Of course, it is theoretically possible to obtain very high accuracy in the relative motion by subtracting extremely accurate absolute positions. However, the accuracy requirement for the cluster as a whole is generally not that high. For a cluster used as a phased-array antenna, perhaps global accuracy of a few tens of meters might suffice, driven by the pointing and tracking requirement when pointed at the Earth. The relative position of satellites may need to be known to substantially higher accuracy, perhaps to the centimeter level. This requirement is driven by the accuracy criterion of one tenth of the operating wavelength for the cluster to perform as a single coherent antenna. Because the accuracy requirements separate, it would be desirable to separate the estimator into two corresponding parts. We will see that such a separation of the two estimation problems is mostly possible.

Dynamics Model

We define the state vector $\boldsymbol{\mathcal{I}}^T = \{R_i^T, V_i^T\}$ for satellite i in terms of the inertial frame position vector $\boldsymbol{\mathcal{R}}_i$ and velocity vector $\boldsymbol{\mathcal{V}}_i$ The solution[1] can be written for any satellite j as

$$\begin{aligned}\boldsymbol{\mathcal{I}}_j(t) &= \boldsymbol{\mathcal{I}}_{\mathrm{PO},j}(t) + \mathcal{I}_{\mathrm{PO_{pert}},j}(t)\\ &+ E_{j\alpha}(t)(\exp\mathcal{J}t)_{\alpha\beta}\{z_\beta(0)\\ &+ \Lambda_{\beta\gamma\delta}(t)z_\gamma(0)z_\delta(0)\}\\ &+ \mathcal{I}_{\mathrm{modalpert},j}\end{aligned} \tag{1}$$

This expression is written in component notation, where greek subscripts are summed from 1 to 6. Here, $\boldsymbol{\mathcal{I}}_{\mathrm{PO}}(t)$ is the periodic orbit in the inertial frame and $\mathcal{I}_{\mathrm{PO,pert}}$ is the forced solution in the periodic orbit. (The reference orbit is actually periodic in the nodal reference frame, which regresses with the orbital plane.) The first-order modal Floquet solution is the second line, with $E(t)$ being the modal matrix and $\mathcal{J}$ the matrix of Poincaré exponents. (This is actually a Jordan form, because four Poincaré exponents are zero.) The modal state vector $\boldsymbol{z}(0)$ for each satellite contains some of the quantities we wish to estimate. The third line includes the second-order solution in the free motion of the satellite, which will be used if necessary to achieve a desired accuracy. Finally, we have found that it is necessary to include the forced solution in the modes of each satellite, and this is the last line. Because the periodic orbit/Floquet solution includes all zonal harmonics, the principal sources of perturbations are air drag and sectoral/tesseral harmonics.

Equation (1) is actually very similar to the Clohessy–Wiltshire[2] (CW) equivalent. In that case, the periodic orbit would be replaced by a nonprecessing two-body circular orbit, the time periodic modal matrix $E(t)$ would be the constant eigenvector matrix of the CW

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solution, and the Poincaré exponent matrix $\mathcal{J}$ is replaced with the CW system's constant coefficient frequencies. Perturbations to the reference circular orbit $\mathcal{I}_{\mathrm{PO,pert},\, j}(t)$ would now need to include the dominant J_2 effects, as well as all of the other zonal harmonics. These perturbations are already included in the periodic orbit in the Floquet approach. Similarly, J_2 and other gravitational harmonics will perturb the first-order modal solution as well, producing the first-order forced perturbations. In the Floquet version used here, these forced perturbations begin with the tesseral harmonics and air drag, which are far smaller than the large J_2 effects. So, whereas the periodic orbit/Floquet mode approach may not be as familiar, it follows the expected pattern.

The periodic orbit is a special structure and is fully described by only four parameters. These are the right ascension of the ascending node at epoch Ω_0, the orbital period τ, the inclination i_0, and the phase angle of the orbit at epoch ϕ. When perturbations are included, an additional parameter describing the effect of air drag can be added. If we allow this additional parameter to be the ballistic coefficient $B = C_d A/m$ (where C_d is the drage coefficient, A the area, and m the mass of a typical satellite), then the periodic orbit can be written in interial frame coordinates as

$$\boldsymbol{\mathcal{I}}_{\mathrm{PO}} = \mathcal{R}_Z^{(2)^T}\{\mathcal{N}_0[t + (\phi\tau/2\pi);\, \tau, i_0]\} + \mathcal{I}_{\mathrm{PO,pert}}(t, B) \tag{2}$$

Here $\mathcal{N}_0$ is the periodic orbit in the nodal frame of reference, where it acutally closes upon itself. In the inertial reference frame, the periodic orbit processes. The inertial Z-axis rotation matrix transforms from the nodal frame of reference and is given by

$$\mathcal{R}_Z^T = \begin{Bmatrix} \cos(\Omega_0 + \dot{\Omega}t) & -\sin(\Omega_0 + \dot{\Omega}t) & 0 \\ \sin(\Omega_0 + \dot{\Omega}t) & \cos(\Omega_0 + \dot{\Omega}t) & 0 \\ 0 & 0 & 1 \end{Bmatrix} \tag{3}$$

It is repeated twice along the diagonal of the six-dimensional $\mathcal{R}^{(2)}$, and the nodal regression rate $\dot{\Omega}$, determined from the periodic orbit calculation, is itself a function of period τ and inclination i_0. The nodal frame state vector $\boldsymbol{\mathcal{N}}$ explicitly depends on the effective time along the orbit as shown in the linear argument $t + \phi\tau/2\pi$, as well as implicitly on the orbital period and inclination. The particular solution depends on the ballistic coefficient of a typical satellite in the cluster.

The periodic orbit in the nodal reference frame is constructed by solving a boundary value problem at a specified period and inclination. The coordinates are then reduced to Fourier series. This process also supplies the nodal regression rate $\dot{\Omega}(\tau, i_0)$. Partial derivatives of the periodic orbit are also needed for the orbit determination process. By constructing a grid of additional periodic orbits varying either τ or i_0, the partial derivatives $\partial\mathcal{N}_0/\partial\tau$ and $\partial\mathcal{N}_0/\partial i_0$ can also be calculated and reduced to Fourier series. The derivatives $\partial\dot{\Omega}/\partial\tau$ and $\partial\dot{\Omega}/\partial i_0$ are also obtained in this way. The time derivative of the nodal solution $\mathcal{N}_0$ is easily found from the Fourier series for the periodic orbit itself. Of course, partial derivatives with respect to the node at epoch, Ω_0, are obtained from Eq. (3) by trivial means.

The particular solution at the linear approximation can be constructed in the nodal frame of reference without using the modal decomposition[1] and obeys

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{N}_p = A\mathcal{N}_p + \mathcal{R}_Z^{(2)}\begin{pmatrix}\mathbf{0}\\ \boldsymbol{a}_p\end{pmatrix} \tag{4}$$

where A is the system matrix for the periodic orbit and $\boldsymbol{a}_p$ is the perturbing acceleration in the inertial frame, here due to air drag and sectoral/tesseral gravity terms. The particular solution has zero intial conditions. However, we have found it just as simple, and very necessary to directly integrate the equations of motion for a "real" satellite (including air drag and the entire geopotential), minus the equations of motion for the periodic orbit:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{I}_p = \dot{\mathcal{I}} - \dot{\mathcal{I}}_{\mathrm{PO}} \tag{5}$$

Here, $\dot{\mathcal{I}}_{\mathrm{PO}}$ is the time derivative of the periodic orbit in the inertial frame. The advantage of the latter method is that there are no expansions or assumptions made about the size of the satellite cluster itself. Also, linear system (4) is neutrally stable, and any numerical perturbation to the integration causes an exponential divergence of the forced solution from reality. The advantage of Eq. (5) is that it is not subject to these numerical instabilities. The derivative of the particular solution with respect to the ballistic coefficient is easily found by using the fact that the equations of motion for the real satellite $\dot{\mathcal{I}}$ are linear in this quantity. Then

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\mathcal{I}_{\mathrm{pert}}}{\partial B} = \frac{\partial\dot{I}}{\partial B} \tag{6}$$

is numerically integrated in parallel with Eq. (5), starting with zero initial conditions. This supplies all of the information necessary to construct the matrix

$$\frac{\partial\mathcal{I}_{\mathrm{PO}}}{\partial(\tau, i_0, \Omega_0, \phi, B)} \tag{7}$$

necessary for the orbit determination process.

The modal matrix $E(t)$ and Jordan form $\mathcal{J}$ are byproducts of determining the periodic orbit. We note that because the periodic orbit will be iterated as a part of the estimation process, it is necessary to use extreme care to choose eigenvectors of the periodic orbit state transition matrix Φ that are aligned with those of the previous iteration. The definition of the modal variables $\boldsymbol{z}$ depends on keeping a fixed convention for these vectors. Our convention is that z_1 and z_2 describe the eccentricity-like motion in the Floquet solution. The variable z_3 is the in-track displacement of the satellite relative to the cluster center, and its conjugate mode z_4 will force a change in orbital period if it is nonzero. The modal variable z_5 is a change in the orbital node, and its conjugate variable z_6 will force differential precession of the orbit planes if it is nonzero.

We had hoped that an accurate enough dynamics model would not require the addition of a first-order forced solution in the modes. This is not the case for the accuracies we wish to achieve. At 637-km altitude and 57-deg inclination, the modal forced solution builds to more than 60 cm at the end of 1 day, above our 2-cm self-imposed limit. Again, because the underlying linear Floquet system is only neutrally stable, a linearized forced solution was found to be violently numerically unstable over time intervals longer than 1 day. Figure 1 shows such a case, in which residuals in the satellite's relative range grow exponentially after the first half day of tracking data. So instead, we have integrated the difference between the accerlation where each satellite actually is and where it thinks it is according to the linearized model. As with the forced solution in the periodic orbit as a whole, this produces a "nonlinear" particular solution, which does not diverge from reality.

There is another way to look at the treatment of the forced solutions. In essence, we have taken a full numerical integration of each satellite and slipped the periodic orbit/Floquet relative motion model into the middle of it. This is much like an Encke's method numerical integration but without the expansions in small differences. Besides avoiding the violent numerical instabilities of the forced linearized systems, it also allows us to (potentially) achieve the same accuracy as can be expected of a full numerical integration.

Data Sources

For the orbit estimation process, we have simulated several plausible data sources. Ground tracking has not been simulated, because newer options give much higher accuracy. GPS data appear as intertial position vectors on each satellite. That is, the data vectors are

$$\boldsymbol{D}_i^T = (X_i, Y_i, Z_i) \tag{8}$$

The uncertainty in these position components was set to $\sigma = \pm 2$ m, perhaps appropriate for the unencrypted GPS system. In addition, satellites equipped with GPS receivers may also use differential

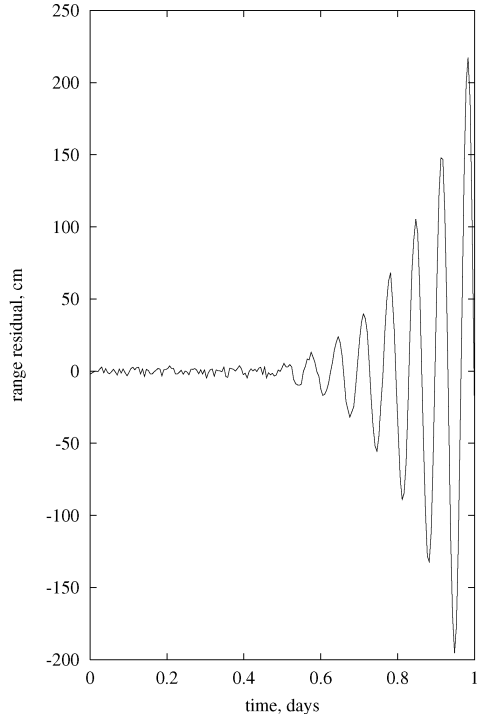

Fig. 1 Residuals in intersatellite range data go exponentially unstable after one half day when a linearized particular solution is used to model perturbations.

techniques, which gives relative position vectors from one satellite to another:

$$\boldsymbol{D}_{ij}^T = (X_i - X_j, Y_i - Y_j, Z_i - Z_j) \tag{9}$$

We have modeled the accuracy of differential GPS data as $\sigma = \pm 2$ cm. This appears to be quite achievable.[3,4] We note that this wide accuracy difference between "absolute" and differential GPS data will shortly cause considerable problems.

Finally, in any phased array or direction finding role, each satellite in the cluster will need an atomic clock for a time reference. Because these will probably be the smaller, less accurate types of atomic clocks, it will be necessary to synchronize them frequently. (This is true even if GPS is in use. GPS receivers contain such "less accurate" atomic clocks.) Sending synchronization pulses from satellite i to satellite j and back allows satellite j to set its clock, but as a byproduct the range between the two satellites becomes known. This produces relative range data:

$$\boldsymbol{D}_{ij} = \left[(X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2\right]^{\frac{1}{2}} \tag{10}$$

Because in the phased-array cluster the satellites must navigate to within a tenth of a wavelength or better (a requirement for any "optical" system), clock synchronization must be at least this accurate. Then one of the advantages of relative ranging data is that their accuracy automatically matches the accuracy required for relative navigation. This is not true for differential GPS data, whose accuracy is dictated by the GPS system itself.

Definitions and Singularities

We have set up a nonlinear least-squares system that attempts to estimate the periodic orbit and modal states of a satellite cluster. Standard techniques produce the least-squares system

$$(T^T Q^{-1} T)\delta \boldsymbol{X} = P^{-1}\delta \boldsymbol{X} = T^T Q^{-1}\boldsymbol{r} \tag{11}$$

Here $\delta\boldsymbol{X}$ are the corrections to the system state, Q is the observation covariance matrix, P is the state covariance matrix, $\boldsymbol{r}$ is the vector of data residuals, and T is the observation sensitivity matrix. A deterministic, batch least-squares estimator was chosen because the requirement to stationkeep individual satellites in the cluster will be minimized only by including the maximum amount of realistic, deterministic dynamics. We suspect that most of the problems reported in this section will also exist for a more traditional circular orbit/Clohessy–Wiltshire dynamics model, too.

At the moment it seems that we have to estimate $6N + 5$ quantities for a cluster with N satellites. This includes six modal variables $\boldsymbol{z}$ for each satellite, as well as the five parameters of the reference periodic orbit. But dynamically this cannot be possible, because N point masses require at most $6N$ variables to specify their states. Of the extra five variables, one is the ballistic coefficient B, presumed to be the same for all of the satellites, and it does not count as a dynamical state. The remaining four states must be superfluous in some fashion. Another way to look at this difficulty is to ask the question, Which periodic orbit best represents the cluster? Any nearby periodic orbit could serve as the reference orbit, but which is the "best" one? We have answered that question by imposing the following four constraints on the cluster:

$$\sum_{i=1}^{N} z_{ik} = 0, \qquad k = 3, 4, 5, 6 \tag{12}$$

In effect, these define the center of the cluster as the point where the cluster has zero average in-track displacement z_{i3}, the same orbital period as the cluster average z_{i4}, the same average node z_{i5}, and the same angular momentum as the cluster average z_{i6}. This definition of the cluster center seems reasonable and even works for "clusters" with only one satellite. However, it does give us a constrained estimation problem. We have handled this by forming the usual inverse of the covariance matrix, and then used Eq. (12) to replace each derivative for elements 3, 4, 5, and 6 of the "last" satellite with their equivalents with respect to the other satellites:

$$\frac{\partial z_{Nk}}{\partial -} = -\sum_{i=1}^{N-1} \frac{\partial z_{ik}}{\partial -}, \qquad k = 3, 4, 5, 6 \tag{13}$$

This produces a constrained estimation problem of order $6N + 1$, where the period, node, and inclination and phase of the periodic orbit replace four modal variables. However, it also means that the periodic orbit and modal states must be estimated in parallel.

There is also a major problem in estimating the modal states from "differential" data. That is, differential GPS and scalar range data contain only information on the relative satellite motion. The differnce between two satellite's relative motion, from Eq. (1), is then

$$\begin{aligned}\mathcal{I}_i(t) - \mathcal{I}_j(t) &= R_z^{(2)}(t)E(t)\exp(\mathcal{J}t)z_i(0) \\ &\quad - R_z^{(2)}(t)E(t)\exp(\mathcal{J}t)z_j(0) \\ &= R_z^{(2)}(t)E(t)\exp(\mathcal{J}t)[z_i(0) - z_j(0)]\end{aligned} \tag{14}$$

This states that only relative modal states can be determined from relative data. Bordner[5] successfully constructed a linear estimator by using only relative data, but it can estimate modal states only with respect to a given satellite, say, satellite 1. Whereas this is quite sufficient to model the relative motion when the cluster is quite small, it is not adequate when the cluster grows large enough that the forced solution becomes appreciable, and it is certainly not sufficient to construct stationkeeping maneuvers: that requires knowledge of the absolute modal states.

At least theoretically, this problem can be solved by including a source of absolute data, either GPS data or ground-based tracking data for each satellite. However, absolute GPS data are two orders of magnitude worse than the relative data we have modeled, and this disparity appears as four orders of magnitude difference in the data covariance matrix Q. This difference has two effects in our problem. First, the combined state covariance matrix P for the periodic orbit plus modal states problem will not invert reliably in double-precision arithmetic. This is a numerical singularity, not a theoretical singularity. So we have formed the joint estimation problem (11) at each iteration and then separately solved the estimation problems for the periodic orbit and the modal variables. This approach partitions the wide disparity in the data accuracy sufficiently that the periodic orbit problem is well defined. Residuals from absolute data are still spread over both the periodic orbit and modal estimation problems at each iteration, but these problems do not wish to be solved together.

Second, notice that the first two modal states, z_1 and z_2, were omitted from Eq. (12), because these variables do change with time, and the Poincaré exponent represents the advance or regression of the line of apsides. However, the motion of the perigee is very small in one orbit, so that the periodic orbit plus these two modes (essentially the eccentricity of the orbit) are very close to being a periodic orbit themselves. (In fact, in the two-body problem, in which the perigee does not move, an eccentric orbit is also periodic.) So, we not only expected another pair of nearly unobservable states from Eq. (14) but were surprised to find a second pair of nearly unobservable states associated with the eccentricity mode states z_1, z_2. This second pair is due to the slow motion of the perigee in one orbit, leading to another source of confusion between what constitutes the periodic orbit and what is relative motion. At the moment we handle this by refusing to make state corrections in the two directions where the singularity appears in the inversion of the covariance P. Although inelegant, this technique is quite practical. This slight difficulty should not be a problem for maneuvers, which mainly involve the states that cause cluster dispersion, namely, z_4 and z_6.

Results

The results we present here are for a cluster of satellites at roughly 637-km altitude, with an orbital inclination of 57 deg. We use the EGM-96 gravity model,[6] sometimes to varying order and degree. The atmosphere is from Regan and Anandakrishran.[7] The truth model was a straightforward numerical integration of all of the individual satellites in intertial rectangular coordinates. At selected times, the "true" state was extracted and data were generated and then corrupted with Gaussian random noise of the specified standard deviation. This formed the input to the estimation algorithm.

The first case shown is 1 day of data, with 200 data points each of absolute and differential GPS data and the same number of range data points for a two-satellite cluster. The cluster is about 1 km in diameter, and the second-order terms from the relative motion solution are not needed. Both the truth model and the estimator are using the EGM-96 gravity model through order 14. (Order 14 is probably not sufficient for an operational system, but by increasing this limit we will report on the effect of mismatched dynamics.) Figure 2 shows final residuals for the absolute GPS data, which contribute mainly to the determination of the periodic orbit. There is no remaining signal in these data; only the expected 2-m random noise remains. The 1 day's worth of data is processed all as one batch, to better make use of the deterministic dynamics of this problem.

Figure 3 shows the residuals in the differential GPS data, and Fig. 4 shows residuals in the single intersatellite range measurement. Again, only the expected 2-cm random noise remains. The fact that only the noise remains in Figs. 2–4 not only verifies that the estimator is functioning correctly but also provide a very strong confirmation that the numerical results can be trusted to very high accuracy.

Figure 5 shows the forced solution in the periodic orbit over 1 day. This is the perturbation to the periodic orbit due to sectoral and tesseral gravity and air drag. The largest component is the in-track displacement due to air drag. However, at the end of 1 day the cross-track component has developed an oscillation with a period of one orbit and an amplitude of over 1 km. Of particular interest here is how much of this perturbed solution can be accounted for by changes in the periodic orbit itself. (To use the Clohessy–Wiltshire circular orbit as a referent, not all perturbations of a circular orbit can be represented by another circular orbit.) So here, we suspect that not all of these perturbations can be accounted for by changing the periodic orbit at the end time. The remainder of these perturbations will need to be absorbed into the modal motion. Of course, all of the satellites in the cluster will receive the same change, but there probably will be changes. The largest perturbation, the in-track motion driven by air drag, can almost certainly be absorbed into a new periodic orbit by changing its period and phase. This problem of handing the previous estimate forward to begin the next estimate is currently a topic of research.

Figure 6 shows the modal forced solution for each of the two satellites. Whereas Fig. 5 showed the effects of sectoral and tesseral gravity and air drag on the periodic orbit, the Fig. 6 shows their effects on the relative motion of the satellites. An in-track drift of

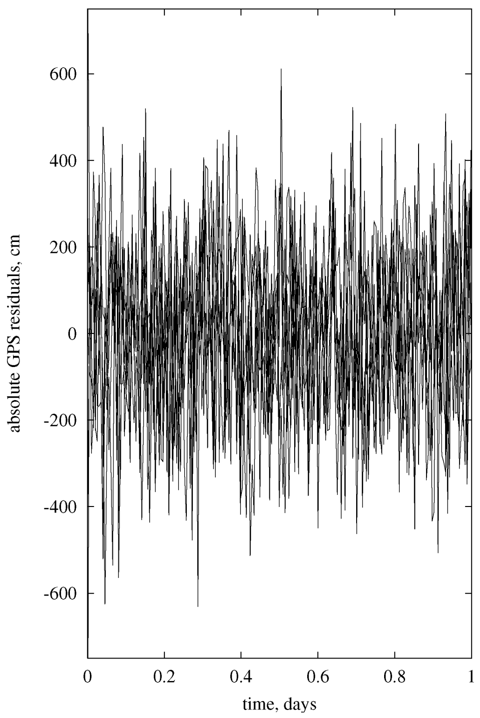

Fig. 2 One day of absolute GPS residuals in the intrack, cross-track, and radial directions. Only the 2-m noise remains.

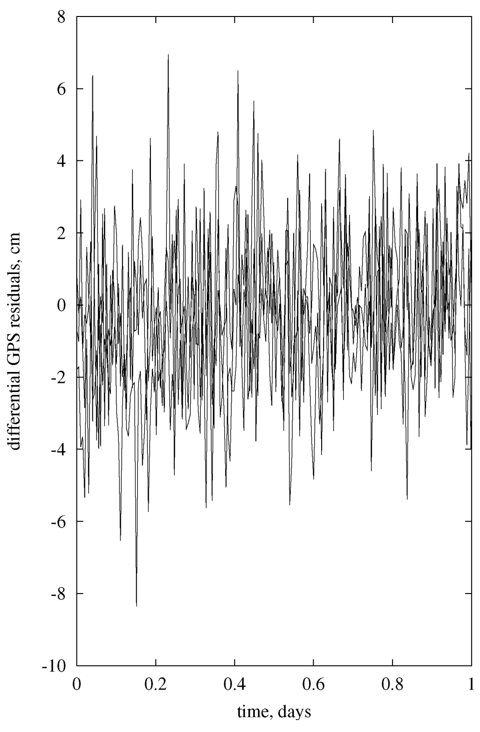

Fig. 3 One day of differential GPS residuals in the in-track, cross-track, and radial directions. Only the 2-cm noise remains.

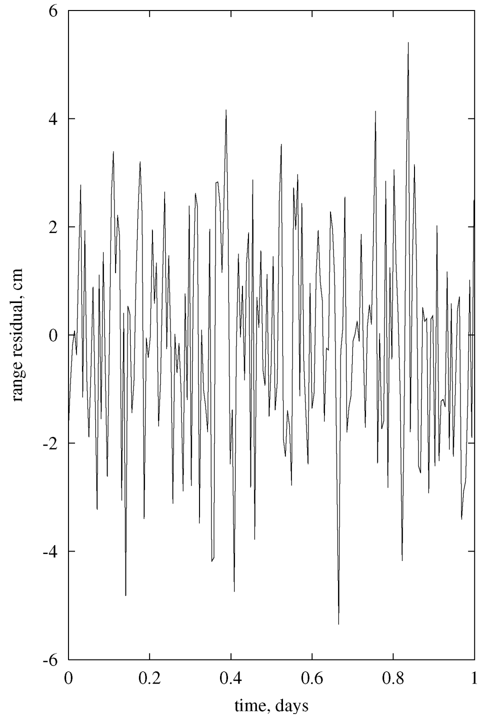

Fig. 4 One day of relative range residuals. Only the 2-cm noise remains.

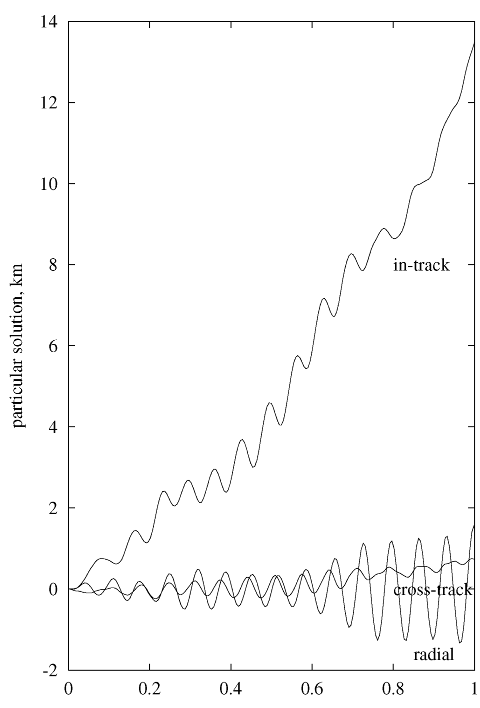

Fig. 5 Forced solution in the periodic orbit over 1 day. The 14-km in-track drift is, of course, due to air drag.

each satellite is again the most pronounced effect, amounting to 5 m over 1 day, or two meters with respect to each other. The small size of these forced solutions shows clearly the advantage of the periodic orbit/Floquet approach, because the only remaining perturbations in the system are due to sectoral, tesseral, and (relative) air drag perturbations. We can also use this plot to contemplate the magnitude of maneuvers necessary to correct the relative drift of the two satellites. If the 2-m relative separation is eliminated over a period of one orbit, the required maneuvers would be of the order of 2 m in 90 min, once per day, or $\Delta v \approx 0.04$ cm/s per

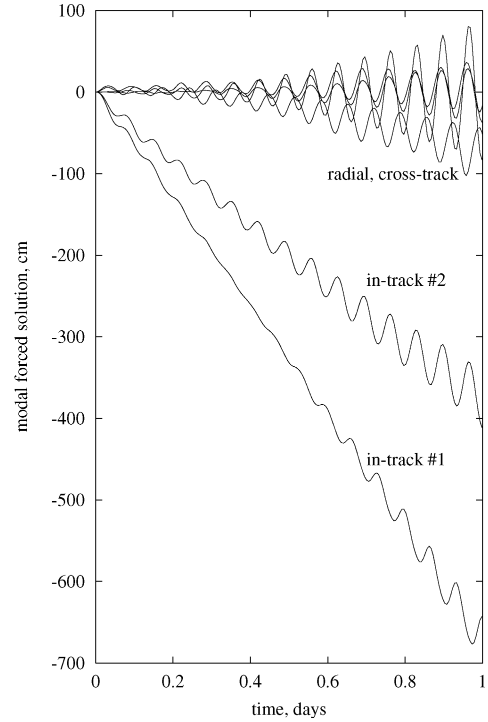

Fig. 6 Forced solution in the modes for two satellites. A 2-m in-track separation has appeared over 1 day.

day. The accuracy of the periodic orbit/Floquet mode approach allows one to clearly discriminate between benign perturbations and those that cause the cluster to disperse. Maneuvers in the periodic orbit/Floquet theory model have been discussed more completely in Wiesel.[8]

Because we are using full nonlinear particular solutions, these results are, in a sense, expected. The periodic orbit/Floquet reference solution has been imbedded into what amounts to a full numerical integration of the equations of motion. The usefulness of the model is best judged by what it does not directly account for: the magnitude of the forced solutions in the periodic orbit and in the modal variables.

The preceding results are all for cases in which the truth model and the estimator have exactly the same dynamics. In the next case, the gravity model used by the truth model was extended to order and degree 35, whereas the estimator continued to use only gravity harmonics through order 14. Figure 7 shows the converged residuals in the absolute GPS data. The best fit periodic orbit clearly has periodic errors of more than 30 m in all directions. On the other hand, Fig. 8 shows the differential GPS residuals for this same case. Whereas there is a secular trend in the in-track residuals, which go from roughly $+8$ cm to -6 cm during 1 day, the cross-track and radial residuals are within the expected 2-cm error. So, most of the dynamics mismatch has been absorbed in a degraded periodic orbit estimate, and the relative motion estimate has suffered only slightly in the process. Whereas this effect has been expected by many authors, it is extremely good news to observe it in practice. In a phased-array radar application, it is the accuracy of the relative motion that must be preserved to enable the cluster to function. A degraded estimate of the position of the cluster center simply requires that a slightly larger area of the ground must be imaged.

Figures 9 and 10 show the estimator's response to a simulated solar storm. At $t = 0.27$ days into the simulation, the air density in the truth model was suddenly increased by a factor of five. The estimator, of course, does not model this effect, because the estimator dynamics have not been informed of the solar flare. The residuals in the absolute GPS data show 130-m errors, mainly in the in-track direction. Clearly the best fit periodic orbit is quite a poor fit and shows the results of trying to minimize the weighted squared residuals by finding a median (and incorrect) drag factor. On the other

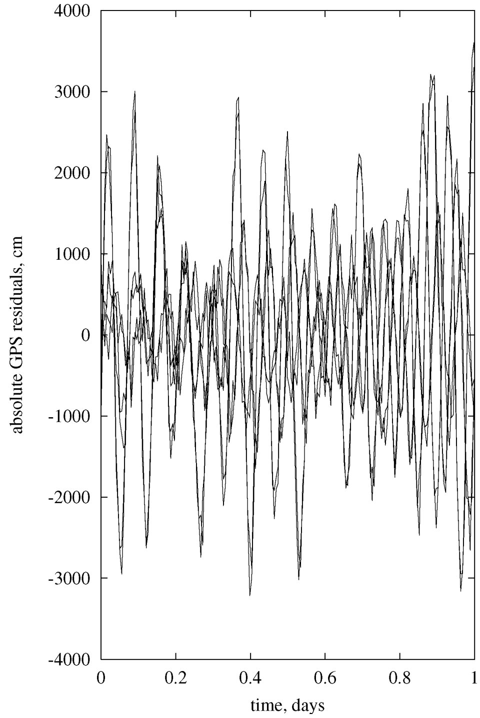

Fig. 7 Absolute GPS residuals for a case with mismatched gravity models. Clear periodic oscillations of several-tens-of-meters amplitude exist in the periodic orbit.

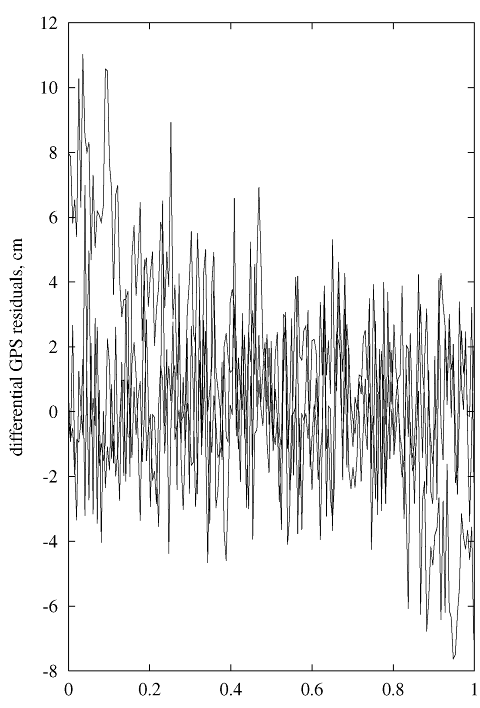

Fig. 8 Converged differential GPS residuals for a case with mismatched gravity models. There is a 10-cm trend in the in-track residuals, but the relative estimate is still quite good.

hand, the relative motion is still very well estimated. Again, this insensitivity of the relative motion to outside perturbations has been predicted by several authors. It is hoped that an alert human analyst would be aware that a solar event had occurred and by segmenting the data stream could obtain much better results for the periodic orbit than are shown in Fig. 9. However, it is the relative estimate that will be critical for cluster operation, and that is not degraded at all.

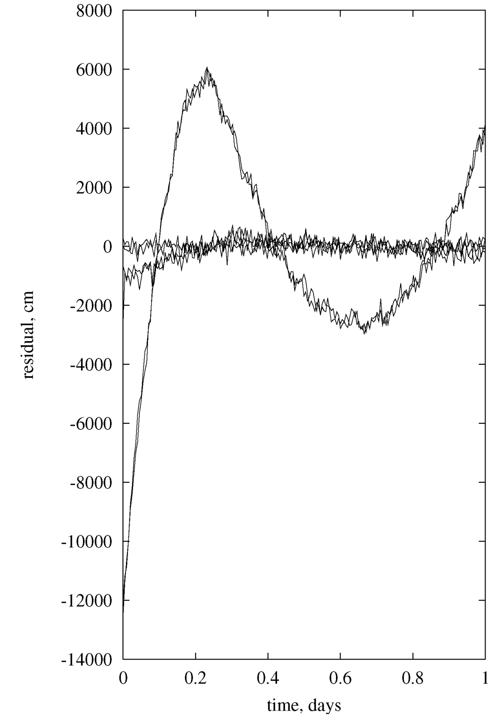

Fig. 9 Absolute GPS residuals for a data stream including a solar flare at $t = 0.27$ days. The in-track residuals are strongly biased, because the estimator model does not include flares.

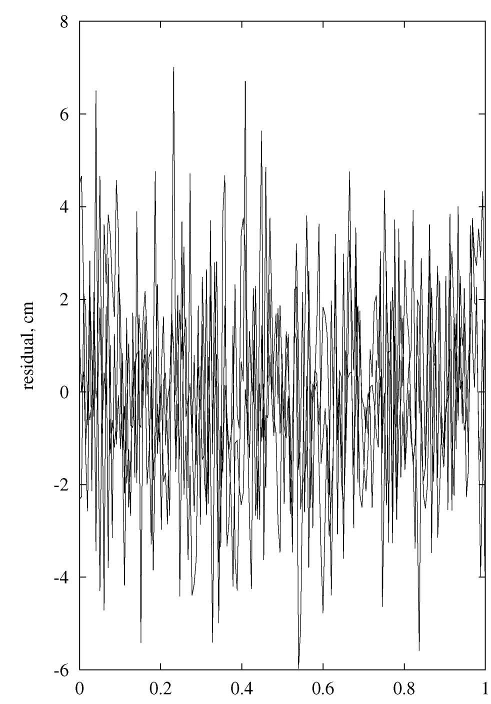

Fig. 10 Converged differential GPS residuals for the solar flare case. The relative motion estimate is not degraded in the slightest.

Conclusions

We have demonstrated that the periodic orbit/Floquet modal model of satellite relative motion can be embedded in a full dynamics estimation model and that results can preserve the full accuracy present in the data. Numerous singularities and numerical difficulties must be carefully handled to achieve this result. In the presence of dynamics model mismatch, it seems that the reference orbit absorbs most of the inaccuracies, whereas the satellite relative motion estimate degrades much less. This is due to the inherent accuracy of the relative motion model, which already includes most of the significant perturbations present in a real Earth-orbiting satellite cluster, and also to the expected insensitivity of the relative motion to disturbing forces. Precise knowledge of the relative and absolute satellite motions will make it possible to precisely stationkeep satellites, greatly reducing the required fuel use.

One outstanding problem is to take a current estimate and move forward to the next batch of data. The forced solutions in both the reference periodic orbit and relative motion solutions must be incorporated into the beginning estimates for the next batch of data, so that their forced solutions can begin at the traditional value of zero. It is expected that not all forced perturbations of the periodic orbit can be described as another periodic orbit (just as not all perturbations of the Clohessy–Wiltshire circular orbit will result in circular orbits).

We also note that one of the biggest difficulties with employing the differential GPS technique is the integer estimator that must be used to insert the number of whole carrier wavelengths between each satellite pair: the so-called "widelane" or "LAMBDA" ambiguity. Our estimator produces results accurate enough that this step can be eliminated, because the differential GPS uncertainty of 19 cm is well above the ±2-cm results reported here.

Current efforts are concentrating on accurate maneuver theories and on the construction of a closed-loop control testbed, in which the entire dynamics, estimation, and control process can be simulated.

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